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The parametric instability of the cyclotron radiation in the absence of resonant particles

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The effect of the simultaneous amplification of two waves due to their parametric cyclotron interaction with modulated ensemble of nonresonant particles is investigated. The mechanism of energy exchange between field and particles in this effect is analyzed.

1. Introduction

This work continues theoretical investigations of unusual regimes of generation of coherent radiation by ensembles of classical charged particles started a few years ago. In these new regimes the simultaneous amplification of two HF waves is provided by their parametric cyclotron interaction with modulated electron ensemble which is stable against generation of these waves separately. The main interest was attracted by so-called maser without inversion (MWI) [1]-[4],-classical analog of quantum effect of inversionless amplification [5].

This paper is devoted to the investigation of another interesting regime of such parametric instability. It is the amplification of bichromatic cyclotron radiation in the absence of resonant particles [2]. The second regime in contrast to the first one where the amplification mechanism corresponds to the parametric interaction with modulated *active* susceptibility can be named as the amplification in *reactive* medium.

2. The model

The effect of amplification of coherent radiation by ensemble of nonresonant particles was revealed in paper [2]. In that particular scheme two Brilluin components of the waveguide mode with the same transverse structure (with respect to the constant magnetic field $\mathbf{B} = \mathbf{z}_0 B_0$) and different (but close) frequencies $\mathbf{E} = \mathbf{y}_0 \sum_{j=1}^{2} ReE_j exp\left(ik_\perp x + ik_{||j}z - i\omega_j t - i\frac{\pi}{2}\right)$, that are resonant to the electrons with momentum components $p_{||}^R$, $p_\perp^R$ at the first harmonic of the cyclotron frequency $\omega_j = eB_0/mc\gamma_R + k_{||j}v_{||}^R$, interact with ensemble of electrons with momentum components close (but not equal) to the resonant values. The electron ensemble is described by the unperturbed distribution function, modulated on the longitudinal coordinate at the initial moment providing the parametric coupling of HF waves

$$f = f_0\left(p_{||}, p_\perp^2/2\right) + f_M\left(p_{||}, p_\perp^2/2\right) cos\left(\varphi_M + \left(k_{||1} - k_{||2}\right) z\right).$$

The analysis in [2] based on linearised kinetic equation in truncated variables has shown that due to the specific dependence of synchronism detunings on momentum which takes into account both relativistic cyclotron detuning and Doppler shift:

$$\Delta_j = \omega_j - eB_0/mc\gamma - k_{||j}v_{||},$$

it is possible to set such modulation of distribution function, that Δ_1 and Δ_2 oscillate in opposite phase. The dependence $f_M\left(p_{||}, p_\perp^2/2\right)$ must be the antisymmetric function of $\left(p_{||} - p_{||}^R\right)$. As consequence the oscillations of corresponding susceptibilities (medium responses on the first and second field) will be "antiphase". Then if all particles are out of the resonance with waves (there is no partial synchronism), i.e.

$$\Delta_j\left(p_{||}, p_\perp\right) t \gg 1 \tag{1}$$

but there is parametric synchronism

$$\left(k_{||1} - k_{||2}\right)\left(v_{||} - v_{||}^R\right) t \ll 1 \tag{2}$$

the parametric coupling of two waves will assume their simultaneous amplification. The linear increment of such amplification is found in [2].

3. The mechanism of energy exchange

The results of previous analysis [2] do not make clear the mechanism of energy exchange between bichromatic field and "nonresonant". Note that this effect is accompanied by amplification of two waves simultaneously but not by scattering from the field of one frequency to another, as in standard induced scattering. In order to clarify the mechanism of energy exchange in investigated process we solve nonlinear equations of particle motion going to the accompanying frame of references where the frequencies of two waves are equal. Note that in this system the standard induced scattering is accompanied by no energy exchange with the medium.

Consider two circulary polarized waves propagating along the constant magnetic wave. The electric and magnetic field can be written as:

$$\mathbf{E} = Re\mathbf{e}_+ E(z) e^{-i\omega t}, \mathbf{B} = Re\mathbf{e}_+ B(z) e^{-i\omega t} + B_0 \mathbf{z}_0$$
$$E(z) = E_0\left(e^{ikz} + e^{-i\varphi - ikz}\right), \mathbf{e}_+ = \mathbf{x}_0 + i\mathbf{y}_0.$$

Consider the relativistic equations of particle motion in this field

$$\dot{\mathbf{p}} = Re\left(-eE(z)\mathbf{e}_+ e^{-i\omega t} - \frac{e}{c}\left[\mathbf{v}, B(z)\mathbf{e}_+ e^{-i\omega t} + B\mathbf{z}_0\right]\right)$$
$$\dot{\mathbf{r}} = \mathbf{v} = \frac{\mathbf{p}}{m\gamma}.$$

Suppose that momentum components of relativistic particles are close to the resonant values so that resonant parameter is large:

$$R = |v_\perp|^2 \omega / c^2 \Delta_0 \gg 1,$$

and Doppler shift is much smaller that cyclotron detuning:

$$k v_{||} \ll \Delta_0,$$

where $\Delta_0 = \omega - eB_0/mc\gamma$.

Since the electron is not resonant to the field (condition (1)), the electron makes large number of oscillations in the wave, so its motion can be presented as superposition of slow part and term oscillating in the wave field: $\mathbf{p} = \left(P_{||} + \tilde{p}_{||}\right) \mathbf{z}_0 + Re\left(P_\perp + \tilde{p}_\perp\right) \mathbf{e}_+ exp\left(-i\left(eB_0/mc\gamma_0\right)t\right)$.

We develop the theory of perturbations with respect to the wave amplitude, which is set to be rather small so that $\omega \left|\tilde{v}_{||}\right| / \Delta_0 c \ll 1$. We finally obtain the expression for the evolution of electron energy averaged over large number of oscillations in wave field in the square approximation:

$$\frac{d}{dt}\langle w \rangle = \frac{1}{2m\gamma_R}\left(\frac{d}{dt}|P_\perp|^2 + \frac{d}{dt}\left\langle |\tilde{p}_\perp|^2 \right\rangle + \frac{d}{dt}P_{||}^2 + \frac{d}{dt}\left\langle \tilde{p}_{||}^2 \right\rangle\right).$$

It can be found that the change of transverse energy is $4\omega/\Delta_0 >> 1$ times larger than change of longitudinal energy. In tern averaged transverse energy consists of energy of gyrorotations $w_\perp^{gr}$ and averaged oscillatory energy $w_\perp^{osc}$. Their time derivatives

$$\frac{d}{dt} w_\perp^{gr} \propto \left(-R^2 + 2R\left|\frac{V_{\perp 0}}{c}\right|^2 - 2R\right)\left(V_{||0}\frac{\partial |E|^2(Z(t))}{\partial Z}\right)$$
$$\frac{d}{dt}\left\langle w_\perp^{osc} \right\rangle \propto \left(R^2 - 2R\left|\frac{V_{\perp 0}}{c}\right|^2 - 2R\right)\left(V_{||0}\frac{\partial |E|^2(Z(t))}{\partial Z}\right)$$

approximately compensate each other. The remainder of this compensation defines the change of full electron energy:

$$\frac{d}{dt}\langle w \rangle = -\frac{\omega}{\Delta_0^3}\frac{e^2}{m\gamma_0}\frac{|V_{\perp 0}|^2}{c^2}\left(V_{||0}\frac{\partial}{\partial Z}|E|^2(Z(t))\right) \qquad (3)$$

The sign of energy change is constant if $kV_{||}t < \pi$ (condition (2)). This sign depends on the sign of synchronism detuning Δ_0. If $\Delta_0 > 0$ the energy of particle decreases if it moves in direction of stronger field. In this case the decrease of energy of gyro-rotations prevails over increase of averaged oscillatory energy. If $\Delta_0 < 0$ the situation is opposite.

Now the role of modulation of electron ensemble on the longitudinal momentum becomes obvious. If at every position Z electrons moves with the same longitudinal velocity number of electrons receiving and losing energy are equal. But setting the initial modulation on z of longitudinal velocity of electrons in ensemble in correspondence with spatial dependence of field amplitude, so that $\varphi_0 = \varphi_M + \frac{\pi}{2} + \pi Sign\left(\Delta_0\right)$, the decrease of the energy of electron ensemble due to its interaction with nonresonant wave field is obtained.

Note that the expression (3) is correct for relativistic particle, moving in the field of standing wave and constant magnetic field. In the absence of magnetic field the motion of nonrelativistic particle is accompanied by the conservation of its full energy. So in this system this mechanism of parametric amplification can not be realized. But if magnetic field is not zero the full energy of nonrelativistic electron changes in correspondence with conservation law for the sum of kinetic energy and the energy of interaction between magnetic moment of oscillating in wave field electron and magnetic field. So the effect of amplification without resonant particles is realized also in this more simple situation. But the analysis of this effect for different ensembles of electrons has shown that the presented here system with relativistic particles close to resonance seems to be the most perspective for possible applications, since it is important to compare the energy contributed by electrons to the amplified HF radiation with energy input necessary for preparation of initial modulation.

4. Conclusion

In the conclusion let us underline the principal difference between the investigated regime of parametric amplification "without resonance" and the standard induced scattering. It is important that in the considered here effect the time of interaction must be restricted by condition (2), i.e. all particles must stay in the parametric synchronism with the waves during interaction. The condition (2) in particular leads to breaking of the low of conservation of photon number in the interaction process, i.e. to the breaking of the Manley-Rowe relation, which forbids such amplification process at the infinite time of interaction.

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5. References

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